What you'll Learn About
What a geometric series is and whether or not the series converges or diverges
The nth term test for divergence

$S_1 = 4$
$S_2 = 12$
$S_3 = 28$
$S_4 = 60$

Given the first 4 terms of the Geometric Series: a) Write the general term of the series, b) Write the power series and c) Find the sum of the series, if possible

General Term — Power Series

A) $4+8+16+32+\cdots+4(2)^n+\cdots = \sum_{n=0}^{\infty} 4(2)^n$ $\quad n=0$

$4+8+16+32+\cdots+2(2)^n+\cdots+4(2)^{n-1} = \sum_{n=1}^{\infty} 4(2)^{n-1}$ $\quad n=1$

Sum Diverges

$S_1 = 1$
$S_2 = \frac{3}{4} = .75$
$S_3 = .8125$
$S_4 = .796$
$S_5 = .8007$
$S_6 = .7998$

B) $1-\frac{1}{4}+\frac{1}{16}-\frac{1}{64}+\cdots - 1\left(-\frac{1}{4}\right)^n+\cdots = \sum_{n=0}^{\infty}(-1)^n\left(\frac{1}{4}\right)^n$

$1-\frac{1}{4}+\frac{1}{16}-\frac{1}{64}+\cdots - 1\left(-\frac{1}{4}\right)^{n-1}+\cdots = \sum_{n=1}^{\infty}(-1)^{n-1}\left(\frac{1}{4}\right)^{n-1}$

Sum Converges to .8 $\quad r=-\frac{1}{4}$

Geometric Series

$\sum_{n=0}^{\infty} a(r)^n$

$a \rightarrow$ starting value

$r \rightarrow$ common ratio

$\sum_{n=1}^{\infty} a(r)^{n-1}$

what you are mult by

C) $5+15+45+135+\cdots+5(3)^n+\cdots = \sum_{n=0}^{\infty} 5(3)^n$

$5+15+45+135+\cdots+5(3)^{n-1}+\cdots = \sum_{n=1}^{\infty} 5(3)^{n-1}$

Sum Diverges base > 1 $\quad |r|>1$

D) $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\cdots - (-1)^n\left(\frac{1}{2}\right)^n+\cdots = \sum_{n=0}^{\infty}(-1)^n\left(\frac{1}{2}\right)^n = \frac{2}{3}$

$1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\cdots-(-1)^{n-1}\left(\frac{1}{2}\right)^{n-1}+\cdots = \sum_{n=1}^{\infty}(-1)^{n-1}\left(\frac{1}{2}\right)^{n-1} = \frac{2}{3}$

$r=-\frac{1}{2}$

Converge $|r|<1 \rightarrow -1<r<1$

Sum $= \frac{a}{1-r}$ $\quad = \frac{1}{1-\left(-\frac{1}{2}\right)} = \frac{1}{\left(\frac{3}{2}\right)}$

Given the first 4 terms of the Geometric Series; a) Write the general term of the series, b) Write the power series, c) Find the equation for the sum of the series, and d) the Interval and Radius of Convergence of the series.

A) $1+5x+25x^2+125x^3+\cdots$

d) $-1 < r < 1$

$-\frac{1}{5} < \frac{5x}{5} < \frac{1}{5}$

($n=0$) general term: $(1)(5x)^n$

power series: $\sum_{n=0}^{\infty}(5x)^n$

I.O.C: $-\frac{1}{5} < x < \frac{1}{5}$

Sum $= f(x) = \frac{1}{1-5x}$

R.O.C: $\frac{1}{5}$

B) $1-(x-1)+(x-1)^2-(x-1)^3+\cdots+(-1)^n(x-1)^n+\cdots=\sum_{n=0}^{\infty}(-1)^n(x-1)^n$

general — Power Series

$r = -(x-1)$

Sum $= f(x) = \frac{1}{1-(-(x-1))} = \frac{1}{1+x-1} = \frac{1}{x}$

I.O.C: $-1 < r < 1$

$-1 < -(x-1) < 1$

$-1 < -x+1 < 1$

$-1 \quad -1 \quad -1$

$-2 < -x < 0$

$\frac{-2}{-1} < \frac{-x}{-1} < \frac{0}{-1}$

$2 > x > 0$

$0 < x < 2$

R.O.C: 1